The Effects of Aggregate Demand Composition on the Economic Growth in Aging Societies

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1. Summary of the Research

(1) Data

① Population

② Service Sector's Share in GDP

③ Sectoral Productivity

④ Elder People's Preference

* food at home (?)
* food at restaurants (?)
* durable goods (?)
* home and health related services (?)
    # service charges for home repairs and maintenance
    # domestic services
    # services related to clothing, medical services, public transportation, recreational services, personal care services

(2) Motivation (Abstract)

Many industrialized countries experience an accelerating aging process and an expansion of the share of service sector in GDP (Petty-Clark's Law). If older people favor services over manufacturing goods, the ongoing demographic change will enhance the expansion of service sector. As a result, the GDP growth will slow down because the labor-productivity of service sector seems to be lower than manufacturing sector.

→ Analysis of two-sector OLG models to investigate the effects of changes in demographic structure on

* economic growth (aggregate variables and per-capita variables)
* industrial structure
* welfare level (utility of each generation)

(3) Assumptions of the Model

① Two-sector (manufacturing sector and service sector), two-period life OLG model.

② Increasing old-to-young population ratio.

③ Each individual favors manufacturing goods over services when "young", and favors services over manufacturing goods when "old".

④ Each individual works (inelastic labor supply) and makes consumption-saving decision when young, and consumes principal and interest income when old.

⑤ Exogenous TFP growth in each sector. TFP growth in manufacturing sector is faster than service sector.

⑥ Service sector is labor-intensive, and manufacturing sector is capital intensive.

⑦ Each sector uses capital and labor, Manufacturing sector produces manufacturing goods and investment goods for capital accumulation. Service sector produces services.

(4) Main Findings

① General Properties

As old/young population ratio increases;

(i) $k \equiv K/N$ (↑) → The growth rates of per-capita variables increase, and the utility of each successive generations improves.

(ii) In general, the service sector's share in GDP increases, and the growth rates of aggregate variables decrease.

② Numerical Analysis

4-major forces affecting the GE resource allocation

(i) The speed of aging process ($n(t) \equiv [N(t+1)/N(t)] - 1$);

Faster aging process → Faster service sector expansion

(K and N shift from manufacturing sector to service sector.)

(ii) The elasticity of substitution between manufacturing goods and services;

* Higher substitutability between manufacturing goods and services → Balance between manufacturing goods and services matters less, while the quantity of either item matters more → K and N shift to manufacturing sector to generate faster quantitative expansion.

* Lower substitutability → Balance between manufacturing goods and services is more important.

(iii) Inter-temporal elasticity of substitution between young-consumption ($C^1(t)$) and old-consumption ($C^2(t+1)$);

* <u>Lower</u> <u>substitutability</u> between $C^1(t)$ and $C^2(t+1)$ → K and N shift from manufacturing sector to service sector to generate an offsetting expansion of service sector. Otherwise, $C^2(t+1)$ becomes too large relative to $C^1(t)$ from the view point of "consumption smoothing". (Service sector's TFP growth is slower than manufacturing sector.)

* <u>Lower</u> <u>substitutability</u> between $C^1(t)$ and $C^2(t+1)$ → <u>Balance</u> between $C^1(t)$ and $C^2(t+1)$ is <u>more</u> important than <u>quantitative expansion</u> of $C^1(t)$ and/or $C^2(t+1)$.

(iv) The elasticity of factor substitution between K and N;

Due to the TFP growth (and the increase in old/young ratio),

$$k \equiv K/N\ (\uparrow) \rightarrow w/r\ (\uparrow)$$

* If the factor substitutability in manufacturing sector is higher than service sector, the pressure to substitute capital for labor is stronger in manufacturing sector than service sector → K shifts from service sector to manufacturing sector, and N shifts from manufacturing sector to service sector. (K/N ratio in manufacturing sector increases relative to service sector.)

2. Details

(1) Model

① Firms (Two Sectors)

* a-sector (manufacturing sector)

$$\max_{\{K_a(t),\, N_a(t)\}} \Pi_a \equiv Y_a(t) - r(t)\,K_a(t) - w(t)\,N_a(t)$$

where

$$Y_a(t) = A(t)[\,\alpha\,(K_a(t))^{1-\eta} + (1-\alpha)(N_a(t))^{1-\eta}\,]^{1/(1-\eta)}$$

$1/\eta$: the elasticity of substitution between K_a and N_a

* b-sector (service sector)

$$\max_{\{K_b(t),\, N_b(t)\}} \Pi_b \equiv p(t)\,Y_b(t) - r(t)\,K_b(t) - w(t)\,N_b(t)$$

where

$$Y_b(t) = B(t)[\,\beta\,(K_b(t))^{1-\gamma} + (1-\beta)(N_b(t))^{1-\gamma}\,]^{1/(1-\gamma)}$$

$1/\gamma$: the elasticity of substitution between K_b and N_b

* Assumptions

(i) Exogenous TFP growth

$g_a(t) \geq g_b(t)$ where

$g_a(t) \equiv [A(t+1)/A(t)] - 1$ and $g_b(t) \equiv [B(t+1)/B(t)] - 1$.

(ii) $\alpha \geq \beta$ (a-sector's capital intensity is higher than b-sector.)

② Households (Two-period Life)

$$\max \quad U(C^1(t), C^2(t+1)) = \frac{[C^1(t)]^{1-s} - 1}{1-s} + d\,\frac{[C^2(t+1)]^{1-s} - 1}{1-s}$$

subject to

$$c_a^1(t) + p(t)\,c_b^1(t) + s(t+1) = w(t)$$

$$c_a^2(t+1) + p(t+1)\,c_b^2(t+1) = (1 + r(t+1) - x\,)\,s(t+1)$$

where

$$C^1(t) \equiv \{e_1[c_a^1(t)]^{1-r_1} + (1-e_1)[c_b^1(t)]^{1-r_1}\}^{1/(1-r_1)}$$

$$C^2(t+1) \equiv \{e_2[c_a^2(t+1)]^{1-r_2} + (1-e_2)[c_b^2(t+1)]^{1-r_2}\}^{1/(1-r_2)} \quad .$$

$1/\sigma$: inter-temporal elasticity of substitution

between $C^1(t)$ and $C^2(t+1)$

$1/\rho_1$: elasticity of substitution between $c_a^1(t)$ and $c_b^1(t)$

$1/\rho_2$: elasticity of substitution between $c_a^2(t+1)$ and $c_b^2(t+1)$

ξ : capital stock depreciation rate

* Assumptions

(i) $\varepsilon_1 \geq \varepsilon_2$: Each individual favors a-type good when young, and favors b-type good when old.

(ii) Inelastic labor supply

(iii) Exogenous population growth rate $n(t) \equiv [N(t+1)/N(t)] - 1$.

③ Market Clearing Conditions

(i) Capital Market

$N(t-1)s(t)$: aggregate supply

$K_a(t) + K_b(t) \equiv K(t)$: aggregate demand

Capital Market Clearing Condition

$K_a(t) = l(t)K(t)$ and $K_b(t) = (1- l(t))K(t)$

where $l(t) \in [0, 1]$.

(ii) Labor Market

$N_a(t) = m(t)N(t)$ and $N_b(t) = (1-m(t))N(t)$

where $m(t) \in [0, 1]$.

(iii) a-type good market

$$Y_a(t) = N(t)c_a^1(t) + N(t-1)c_a^2(t) + K(t+1) - (1-x)K(t) \ .$$

(iv) b-type good market

$$Y_b(t) = N(t)c_b^1(t) + N(t-1)c_b^2(t)$$

④ GE Dynamical System

Given the exogenous process { $A(t)$, $B(t)$, $N(t)$; t = 0, 1, 2, ... } and $k(0)$ ≡ K(0)/N(0), the general equilibrium is described by the following system of difference equations with respect to { $l(t)$, $m(t)$, $p(t)$, $k(t+1)$; t = 0, 1, 2, ... }.

* a-sector's MRTS = b-sector's MRTS

$$F_1(\ l(t), m(t), p(t), k(t)\ ) = \left(\frac{1-a}{a}\right)\left(\frac{l(t)k(t)}{m(t)}\right)^h - \left(\frac{1-b}{b}\right)\left(\frac{(1-l(t))k(t)}{1-m(t)}\right)^g = 0$$

* a-sector's MPK = b-sector's MPK

$$F_2(\ l(t), m(t), p(t), k(t)\ ) = A(t)[\,a\,(\,l(t)k(t))^{1-h} + (1-a)\,m(t)^{1-h}\,]^{h/(1-h)}\,a\,(\,l(t)k(t))^{-h}$$
$$-\,p(t)B(t)[\,b\,((1-l(t))k(t))^{1-g} + (1-b)(1-m(t))^{1-g}\,]^{g/(1-g)}\,b\,((1-l(t))k(t))^{-g} = 0$$

* b-type good market clearing condition

$$F_3(\ l(t-1), l(t), l(t+1), m(t-1), m(t), m(t+1), p(t-1), p(t), p(t+1),$$
$$k(t-1), k(t), k(t+1)\ ) = B(t)[\,b\,((1-l(t))k(t))^{1-g} + (1-b)(1-m(t))^{1-g}\,]^{1/(1-g)}$$
$$-\ [1/(1+n(t-1))][(1-e_2)P^2(t)/p(t)]^{1/r_2}(Z^2(t)/P^2(t))$$
$$+\ [(1-e_1)P^1(t)/p(t)]^{1/r_1}(Z^1(t)/P^1(t)) = 0$$

* capital market clearing condition

$$F_4(\ l(t), l(t+1), m(t), m(t+1), p(t), p(t+1), k(t), k(t+1)\ )$$

$$= k(t+1) - \frac{w(t)/(1+n(t))}{1+d^{-1/s}\,[(1-x+r(t+1))P^1(t)/P^2(t+1)]^{(s-1)/s}} = 0$$

(2) Simplified Model

① Firms

* $\eta = \gamma = 1$ (Cobb-Douglas Technologies)

* $0 = \beta < \alpha < 1$ (b-sector uses labor only)

$$Y_a(t) = A(t)\,K(t)^a\; N_a(t)^{1-a} \text{ : a-sector}$$

$$Y_b(t) = B(t)\,N_b(t) \text{ : b-sector}$$

② Households

* $\sigma = 1$ and $\rho_1 = \rho_2 = 1$ (log-utility)

* $\varepsilon_1 = 1$ and $\varepsilon_2 = 0$ (Each individual consumes only a-type good when young, and only b-type good when old.)

$$U(\,c_a^1(t), c_b^2(t+1)\,) = \ln c_a^1(t) + d\,\ln c_b^2(t+1)$$

③ Market Clearing conditions

(i) Capital Market

$$K(t) = N(t-1)\,s(t)$$

(ii) Labor Market

$$N_a(t) = m(t)\,N(t) \quad \text{and} \quad N_b(t) = (1-m(t))\,N(t)$$

(iii) a-type good market

$$Y_a(t) = N(t)\,c_a^1(t) + K(t+1) - (1-x)\,K(t)$$

(iv) b-type good market

$$Y_b(t) = N(t-1)\,c_b^2(t)$$

④ GE Dynamical System

Given the exogenous process { $A(t)$, $B(t)$, $N(t)$; t = 0, 1, 2, ... } and $k(0)$ ≡ K(0)/N(0), the general equilibrium is described by the following dynamical system with respect to { $m(t)$, $k(t+1)$; t = 0, 1, 2, ... }.

(3.4) $(1-x)\,m(t)^{a} = [1-a-m(t)]\,A(t)\,k(t)^{a-1}$

(3.5) $k(t+1) = \left(\dfrac{1}{1+n(t)}\right)\left(\dfrac{d}{1+d}\right)(1-a)\,A(t)\,[k(t)/m(t)]^{a}$

Theorem 1 (Existence of a unique equilibrium path).

For a given exogenous process { $A(t)$, $B(t)$, $N(t)$; t = 0, 1, 2, ... } and $k(0)$ ≡ K(0)/N(0), the dynamical system has a unique equilibrium path { $m(t)$, $k(t+1)$; t = 0, 1, 2, ... }.

Proof of Theorem 1 → Figure 2

Theorem 2 (Existence of a unique Balanced Growth Path).

If [A(t+1)/A(t)] − 1 ≡ g_a and [N(t+1)/N(t)] − 1 ≡ n are constants, then the dynamical system has a unique BGP.

Theorem 3 (Global Stability of BGP).

For any given k(0) > 0, the dynamical system converges to the unique BGP.

Proof of Theorem 2 and Theorem 3;

(i) Define $\hat{k}(t) \equiv [(1+g_a)^{1/(1-\alpha)}]^{-t}\ k(t)$. Then the dynamical system is rewritten as

(3.4)' $(1-x)\,m(t)^{a} = [1-a-m(t)]\,A(0)\,\hat{k}(t)^{a-1}$

(3.5)' $\hat{k}(t+1) = \left(\dfrac{d}{1+d}\right)\left(\dfrac{1-a}{(1+n)(1+g_a)^{1/(1-a)}}\right)[A(0)\hat{k}(t)^{a-1}]\,m(t)^{-a}\,\hat{k}(t)$.

This system has a unique steady state

(3.15) $m_s = (1-a)\{1-(1-x)[d/(1+d)]/[(1+n)(1+g_a)^{1/(1-a)}]\}$

(3.16) $\hat{k}_s = \left[\left(\dfrac{d}{1+d}\right)\left(\dfrac{(1-a)\,A(0)}{(1+n)(1+g_a)^{1/(1-a)}\,m_s^{a}}\right)\right]^{1/(1-a)}$

(ii) As we saw in the proof of Theorem 1, (3.4)' determines a unique μ(t) ∈ (0, 1–α) as a function of $\hat{k}(t)$, μ(t) = M($\hat{k}(t)$). Then (3.5)' is a 1st order difference equation w.r.t. { $\hat{k}(t)$ };

$\hat{k}(t+1)$ = F($\hat{k}(t)$)

where

$$F(\hat{k}(t)) \equiv \left(\frac{d}{1+d}\right)\left(\frac{(1-a)\,A(0)}{(1+n)(1+g_a)^{1/(1-a)}}\right)[\hat{k}(t)/M(\hat{k}(t)]^{a}\ .$$

It can be shown that F(0) = 0, F(∞) = ∞, F'($\hat{k}$) > 0 for $\hat{k}$ > 0, and

$\lim_{\hat{k}\to 0} F'(\hat{k}) = \infty$. Therefore, at the unique $\hat{k}_s > 0$,

0 < F'($\hat{k}_s$) < 1. (See Figure 3.)

Lemma; $m_s(\overset{+}{n},\ \overset{+}{g_a},\ \overset{+}{x},\ \overset{-}{d},\ \overset{-}{a})$

On the BGP

* r(t) is constant
* k(t), w(t), and $c_a^1(t)$ grow at $(1+g_a)^{1/(1-\alpha)}$
* If $[B(t+1)/B(t)] - 1 \equiv g_b$ is constant, $c_b^2(t)$ grows at $1+g_b$ and p(t) grows at $(1+g_a)^{1/(1-\alpha)}/(1+g_b)$.
* $Y_a(t)$ grows at $(1+g_a)^{1/(1-\alpha)}\times(1+n)$
* $Y_b(t)$ grows at $(1+g_b)\times(1+n)$
* GDP $\equiv Y_a(t) + p(t)Y_b(t)$ grows at $(1+g_a)^{1/(1-\alpha)}\times(1+n)$
* The share of b-sector in GDP is constant;

$$p(t)Y_b(t)/GDP = (1-m_s)/[\frac{m_s}{1-a}+1-m_s]$$

⑤ The Effects of a Change in Population Growth Rate on BGP

Notice

$$1+n = \frac{N(t)}{N(t-1)} = \frac{young}{old}$$

n ($\downarrow$) $\Rightarrow$ Higher old/young ratio

Interpretation

(i) Comparison of identical economies having different (constant) population growth rate n.

(ii) Transition of an economy from initial BGP to final BGP in a long-run.

As n decreases;

* μ_s = [$N_a(t)/N(t)$] decreases.
* $\hat{k}_s$ increases due to;
  - # direct effect from n
  - # indirect effect (n $\downarrow \Rightarrow \mu_s \downarrow \Rightarrow \hat{k}_s \uparrow$)
* [$p(t)Y_b(t)$/GDP] increases.
* A change in n affects the level of U(t).
  It does not affect the growth rate of U(t).

Theorem 4.

$$\frac{\partial}{\partial n}U(t) \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \quad \begin{matrix} (\mathit{Paradoxical\ Case}) \\ \\ (\mathit{Normal\ Case}) \end{matrix} \quad \text{as} \quad m_s \begin{pmatrix} > \\ = \\ < \end{pmatrix} \frac{1}{1+d}$$

Intuitive explanation;

When n increases, $\hat{k}_s$ decreases. Then;

w(t) $\downarrow \Rightarrow$ 1st period (young) consumption decreases

r(t) $\uparrow \Rightarrow$ 2nd period (old) consumption increases.

When μ_s is small, $\hat{k}_s$ is large.

w(t) is large $\rightarrow$ young consumption is large

r(t) is small $\rightarrow$ old consumption is small.

When old consumption is small, $\Delta U(t)/\Delta c_b^2(t)$ is large.

(3) Numerical Analysis of the Simplified Model

(3.4) $$(1-x)\, m(t)^{a} = [1-a-m(t)]\, A(t)\, k(t)^{a-1}$$

(3.5) $$k(t+1) = \left(\frac{1}{1+n(t)}\right)\left(\frac{d}{1+d}\right)(1-a)\, A(t)\, [k(t)/m(t)]^{a}$$

Assumptions

$(A(t+1)/A(t)) - 1 = g_a = 0.12$ with $A(0) = 10$: a-sector TFP

$(B(t+1)/B(t)) - 1 = g_b = 0.1$ with $B(0) = 10$: b-sector TFP

$n(t) = (N(t+1)/N(t)) - 1$: population growth rate (See Figure 4.)

Case 1 (Stationary Case) → Index _1

$n(t) = 0.2$ for all t = 0, 1, ... , 40.

Case 2 (Moderate Aging Process) → Index _2

$n(0) = 0.2$, $n(t) = 0.2 - 0.01 \times t$ for t = 1, 2, ... , 20, and $n(t) = 0$ for t = 21, 22, ... , 40.

Case 3 (Rapid Aging Process) → Index _3

$n(0) = 0.2$, $n(t) = 0.2 - 0.02 \times t$ for t = 1, 2, ... , 20, and $n(t) = -0.2$ for t = 21, 22, ... , 40.

$d = 0.8$: the old-utility discount factor

a : Cobb-Douglas coefficient on capital in a-sector

x : capital depreciation rate

	Normal Case	Paradoxical Case
a	0.3	0.1
x	0.1	0.9

Normal Case Simulation

Figure 3A1 { $\mu(t)$ } : The speed of labor-shift from a-sector to b-sector is faster for the rapid aging case.

Figure 3A2 { k(t) } : The speed of k-accumulation is faster for the rapid aging case.

Figure 3A3 { $\hat{k}(t)$ } : Once n(t) becomes constant at t = 20, $\hat{k}(t)$ quickly converges to steady state $\hat{k}_s$ in each case. Smaller long-run n(t) (=0.2) in the rapid aging case results in larger $\hat{k}_s$.

Figure 3A4 { $p(t)Y_b(t)/GDP(t)$ } : The speed of b-sector expansion is faster for the rapid aging case.

Figure 3A5 { $\Delta GDP(t)/GDP(t)$ } : GDP growth is slower for the rapid aging case.

Figure 3A6 (Normal Case : negative correlation b/w n and U(t))
Figure 3B3 (Paradoxical Case : positive corr. b/w n and U(t))

Each successive U(t) improves due to k-deepening.

In the normal case, $U(t) < 1/(1+\delta) = 0.556$ for all t.

In the paradoxical case, $U(t) > 1/(1+\delta)$ for all t.

(4) Numerical Analysis of the General Model

(2.14) $F_1(\ \lambda(t), \mu(t), p(t), k(t)\ ) = 0,\ t = 0, 1, \dots, 79$

(2.15) $F_2(\ \lambda(t), \mu(t), p(t), k(t)\ ) = 0,\ t = 0, 1, \dots, 79$

(2.16) $F_3(\ \lambda(t-1), \lambda(t), \lambda(t+1), \mu(t-1), \mu(t), \mu(t+1),$

$p(t-1), p(t), p(t+1), k(t-1), k(t), k(t+1)\ ) = 0,\ t = 0, 1, \dots, 79$

(2.17) $F_3(\ \lambda(t), \lambda(t+1), \mu(t), \mu(t+1), p(t), p(t+1),$

$k(t), k(t+1)\ ) = 0,\ t = 0, 1, \dots, 78$

Assumptions

* $k(0)$ is set at the initial steady state value.

*$\{\lambda(80), \mu(80), p(80), k(80)\}$ are set at the final steady state values.

* (2.14)~(2.17) are $(4\times80)-1$ equations for $(4\times80)-1$ variables;

$\{\ \lambda(t), \mu(t), p(t);\ t = 0, 1, \dots, 79\ \}$ and $\{\ k(t);\ t = 1, 2, \dots, 79\ \}$.

* Population Growth Rate

$n(t) \equiv [N(t+1)/N(t)] - 1$

Case 1 (Stationary Case) → Index _1

Case 2 (Moderate Aging Process) → Index _2

Case 3 (Rapid Aging Process) → Index _3

(See Figure *.)

* TFP Growth Rates

$g_a(t) \equiv [A(t+1)/A(t)] - 1$ and $g_b(t) \equiv [B(t+1)/B(t)] - 1$

(See Figure 5.)

Attention is paid on the values of variables between t = 20 and t = 40 where $g_a(t) > g_b(t)$ and n(t) is decreasing.

Benchmark Simulation

{ $\sigma = 2$, $\delta = 0.8$, $\varepsilon_1 = 0.75$, $\varepsilon_2 = 0.25$, $\rho_1 = 1$, $\rho_2 = 1$,

$\eta = 1$, $\gamma = 1$, $\alpha = 0.3$, $\beta = 0.2$, $\xi = 0.02$ }

* U(C^1(t), C^2(t+1)) = $\dfrac{[\,C^1(t)\,]^{1-s}\ -1}{1-s} + d\,\dfrac{[\,C^2(t+1)\,]^{1-s}\ -1}{1-s}$

$1/\sigma = 1/2 = 0.5$: elasticity of substitution between C^1(t) and C^2(t+1)

$\delta = 0.8$: subjective discount factor

* $C^1(t) \equiv \{e_1\,[\,c_a^1(t)\,]^{1-r_1} + (1-e_1)\,[\,c_b^1(t)\,]^{1-r_1}\,\}^{1/(1-r_1)}$

$\varepsilon_1 = 0.75$: weight on c_a^1 relative to c_b^1

$1/\rho_1 = 1/1$: elasticity of substitution between c_a^1 and c_b^1

* $C^2(t+1) \equiv \{e_2\,[\,c_a^2(t+1)\,]^{1-r_2} + (1-e_2)\,[\,c_b^2(t+1)\,]^{1-r_2}\,\}^{1/(1-r_2)}$

$\varepsilon_2 = 0.25$: weight on c_a^2 relative to c_b^2

$1/\rho_2 = 1/1$: elasticity of substitution between c_a^2 and c_b^2

$\xi = 0.02$: capital stock depreciation rate

* $Y_a(t) = A(t)[\,a\,(K_a(t))^{1-h} + (1-a)\,(N_a(t))^{1-h}\,]^{1/(1-h)}$

$\alpha = 0.3$: weight on K_a relative to N_a

$1/\eta = 1/1$: elasticity of substitution between K_a and N_a

* $Y_b(t) = B(t)[\,b\,(K_b(t))^{1-g} + (1-b)\,(N_b(t))^{1-g}\,]^{1/(1-g)}$

$\beta = 0.2$: weight on K_b relative to N_b

$1/\gamma = 1/1$: elasticity of substitution between K_b and N_b

Benchmark simulation

Figure 4A1 { $k(t)$ }

Figure 4A2 { $\lambda(t) = K_a(t)/K(t)$ }

Figure 4A3 { $\mu(t) = N_a(t)/N(t)$ }

The behavior of { $\mu(t)$, $k(t)$ } is similar to the simplified model. As old/young ratio increases, K and N shift from a-sector to b-sector. The speed of the shift is faster for the rapid aging case.

Figure 4A5 { $p(t)Y_b(t)/GDP(t)$ }

Figure 4A6 { $\Delta GDP(t)/GDP(t)$ }

GDP growth rate is $[(1+g_a(t))^{1/(1-\alpha)}]\times(1+n(t))$ → The graphs of $\Delta GDP(t)/GDP(t)$ are the vertical multiplication of the graphs of $g_a(t)$ and $n(t)$. (See Figure 5.)

Figure 4A7 and Figure 4A8 { $U(t)$ }

negative correlation between $n(t)$ and $U(t)$ → Normal Case

Test 1. { e_1 **,** e_2 **}**

Figure Test 1-1A ~ Figure Test 1-1F

Stationary Population Growth Case

Figure Test 1-2A ~ Figure Test 1-2F

Rapid Aging Case

{ $\varepsilon_1 = 0.5, \varepsilon_2 = 0.5$ } → Index _1

{ $\varepsilon_1 = 0.75, \varepsilon_2 = 0.25$ } → Index _2

{ $\varepsilon_1 = 1, \varepsilon_2 = 0$ } → Index _3

* As an individual's young preference on a-type good ε_1 and/or old preference on b-type good $1-\varepsilon_2$ increases;

the size and speed of the shift of K and N from a-sector to b-sector increases.

the size and speed of b-sector expansion increases.

these characteristics are intensified in the rapid aging case.

GDP growth rate decreases in the rapid aging case between t = 20 and t = 40.

* Intuitive Reason

In the extreme case { $\varepsilon_1 = 1, \varepsilon_2 = 0$ }, $C^1(t) = c_a^1(t)$ and $C^2(t+1) = c_b^2(t+1)$. Since $g_a(t) > g_b(t)$, c_b^2 will be too small relative to c_a^1 (**consumption smoothing**). → K and N must shift from a-sector to b-sector to generate an offsetting b-sector expansion.

Test 2. { 1/s **}**

Figure Test 2-1A ~ Figure Test 2-1F

Stationary Population Growth Case

Figure Test 2-2A ~ Figure Test 2-2F

Rapid Aging Case

$1/\sigma = 1/2 = 0.5 \rightarrow$ Index _1

$1/\sigma = 1/1 = 1 \rightarrow$ Index _2

$1/\sigma = 1/0.5 = 2 \rightarrow$ Index _3

* When the elast. subst. b/w $C^1(t)$ and $C^2(t+1)$ ($1/\sigma$) is small (large), K and N shift from a-sector to b-sector (from b-sector to a sector), and $p(t)Y_b(t)/GDP(t)$ increases (decreases).

* The factor shift from b-sector to a sector under large $1/\sigma$ is weak in the rapid aging case because of the countervailing force generated by increasing old/young ratio.

Intuitive Reason

* When $1/\sigma$ is large, the substitutability b/w $C^1(t)$ and $C^2(t+1)$ is large. $\rightarrow$ Quantitative expansion of $C^1(t)$ and/or $C^2(t+1)$, which can be done by shifting K and N to fast-growing a-sector, is good for each individual's utility.

* When the substitutability b/w $C^1(t)$ and $C^2(t+1)$ is small, balancing (smoothing) $C^1(t)$ and $C^2(t+1)$ is more important. This is done by shifting K and N to b-sector to offset its slow TFP growth.

Test 3. **{** r_2 **}** (ρ_1 is fixed at 1.)

Figure Test 3-1A ~ Figure Test 3-1F

Stationary Population Growth Case

Figure Test 3-2A ~ Figure Test 3-2F

Rapid Aging Case

$1/\rho_1$: elasticity of substitution between $c_a^1(t)$ and $c_b^1(t)$

$1/\rho_2$: elasticity of substitution between $c_a^2(t+1)$ and $c_b^2(t+1)$

Index 1	Index 2	Index 3
ρ_2 = 0.5	ρ_2 = 1	ρ_2 = 2
$1/\rho_2$ = 2	$1/\rho_2$ = 1	$1/\rho_2$ = 0.5

* When the elast. subst. b/w $c_a^2(t+1)$ and $c_b^2(t+1)$ is large ($1/\rho_2$ = 1/0.5 = 2), K and N shift from b-sector to a-sector, and $p(t)Y_b(t)/GDP(t)$ decreases. When $1/\rho_2$ is small ($1/\rho_2$ = 1/2 = 0.5), K and N shift the opposite direction ($a \rightarrow b$), and b-sector's share in GDP increases.

Intuitive Reason

* When $1/\rho_2$ is large ($1/\rho_2$ = 1/0.5 = 2), the substitutability between $c_a^2(t+1)$ and $c_b^2(t+1)$ is high. $\rightarrow$ Quantitative expansion of $c_a^2(t+1)$ and/or $c_b^2(t+1)$ is good for U(t). Since $g_a(t) > g_b(t)$, K and N shift from b-sector to a-sector to generate the quantitative expansion.
* When $1/\rho_2$ is small ($1/\rho_2$ = 1/2 = 0.5), balance between $c_a^2(t+1)$ and $c_b^2(t+1)$ is more important. $\rightarrow$ K and N must shift from a-sector to b-sector to offset the slow TFP growth of b-sector.

Test 4. { b **}** (α is fixed at 0.3)

Figure Test 4-1A ~ Figure Test 4-1F

Stationary Population Growth Case

Figure Test 4-2A ~ Figure Test 4-2F

Rapid Aging Case

$\beta = 0.3 \rightarrow$ Index _1

$\beta = 0.2 \rightarrow$ Index _2

$\beta = 0.1 \rightarrow$ Index _3

* For a given K/N ratio, when β is large (small), the return on K_b is large (small) and the return on N_b is small (large) in b-sector.

$\rightarrow \lambda(t) = K_a(t)$ is small (large) and $\mu(t) = N_a(t)/N(t)$ is large (small).

* When β is large, K_a/N_a is small. Because investment good is produced by a-sector, and because $g_a(t) > g_b(t)$, k-accumulation is slower.

Test 5. { h, g }

Figure Test 5-1A ~ Figure Test 5-1F

Stationary Population Growth Case

Figure Test 5-2A ~ Figure Test 5-2F

Rapid Aging Case

$1/\eta$: elasticity of substitution between K_a and N_a

$1/\gamma$: elasticity of substitution between K_b and N_b

Index 1	Index 2	Index 3
$\eta = \gamma$ (1) (1)	$\eta > \gamma$ (1) (0.8)	$\eta < \gamma$ (0.8) (1)
$1/\eta = 1/\gamma$ (1) (1)	$1/\eta < 1/\gamma$ (1) (1.25)	$1/\eta > 1/\gamma$ (1.25) (1)

* As k(t) increases due to TFP growth (and decreasing n(t) in the rapid aging case), the factor price ratio w(t)/r(t) increases. Each sector will try to substitute capital for labor.

When $1/\eta = 1 < 1/\gamma = 1.25$, the factor substitution is easier in b-sector than a-sector. → K shifts from a-sector to b-sector ($\lambda(t)$ ↓ as lambda_2) and N shifts from b-sector to a-sector ($\mu(t)$ ↑ as mu_2).

When $1/\eta = 1.25 > 1/\gamma = 1$, $\lambda(t)$ ↑ (as lambda_3) and $\mu(t)$ ↓ (as mu_3).

* Because K_a/N_a is larger when $1/\eta > 1/\gamma$, k-accumulation is faster. (a-sector produces investment good, and $g_a(t) > g_b(t)$.)

3. Remaining Issues

(1) Apply the analysis to CGE models to derive quantitative predictions.

(2) Endogenize TFP growth by rebuilding as endogenous growth models.

(3) Incorporate public sector to investigate the effects of socio-economic policies such as;

* public efforts to affect population growth rate.
* public efforts to improve TFP.
* social security system.

(4) In reality, services as inputs in business sector are important for economic growth, e.g., the role ITC industry in 1990s.